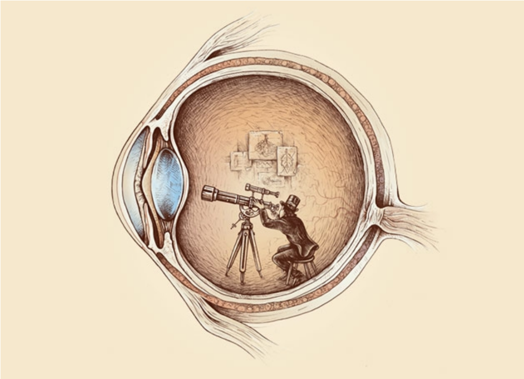


Computer Vision

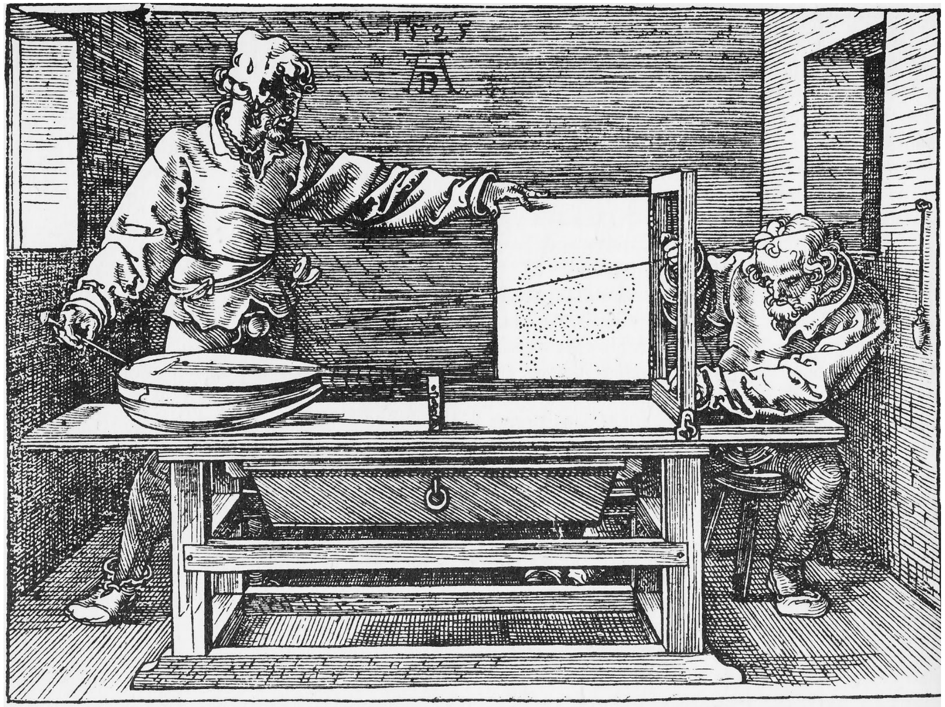
CS 776 Fall 2018

Cameras & Photogrammetry 1

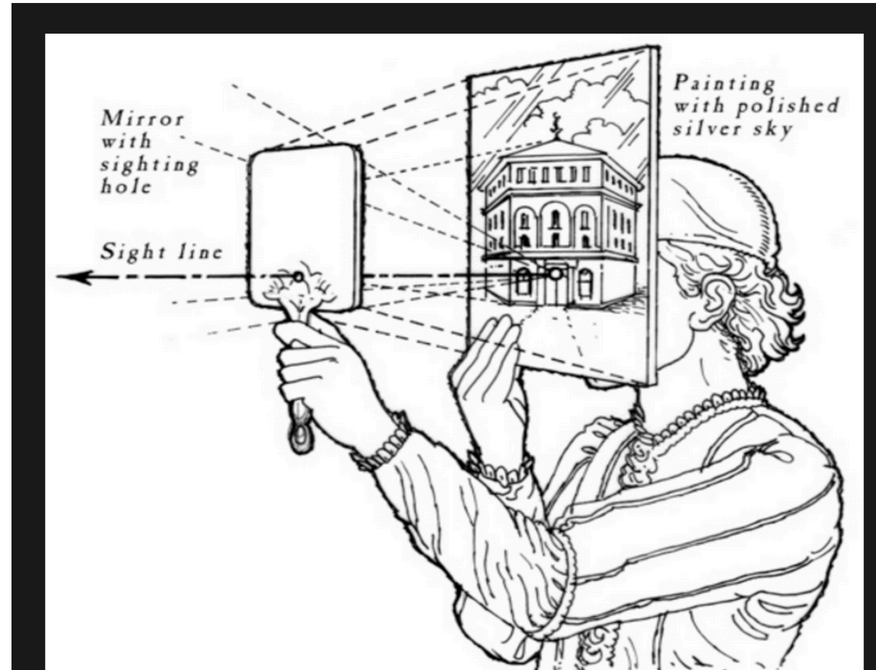
Prof. Alex Berg

(Slide credits to many folks on individual slides)

Cameras & Photogrammetry 1



Albrecht Dürer early 1500s



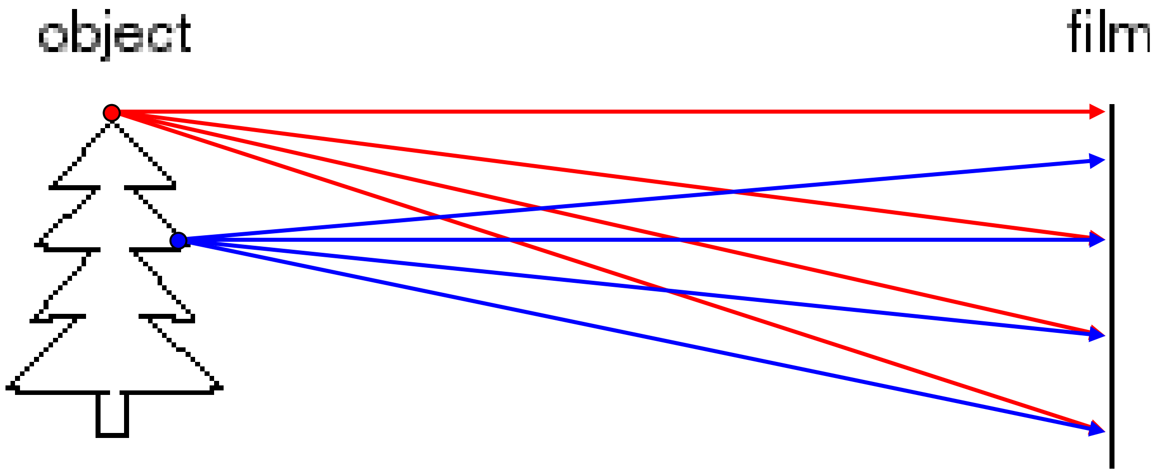
The kind of mirror apparatus described by Brunelleschi in his notebooks...

Brunelleschi, early 1400s

Overview of next two lectures

- The pinhole projection model
  - Qualitative properties
  - Perspective projection matrix
- Cameras with lenses
  - Depth of focus
  - Field of view
  - Lens aberrations
- Digital cameras
  - Sensors
  - Color
  - Artifacts

Let's design a camera

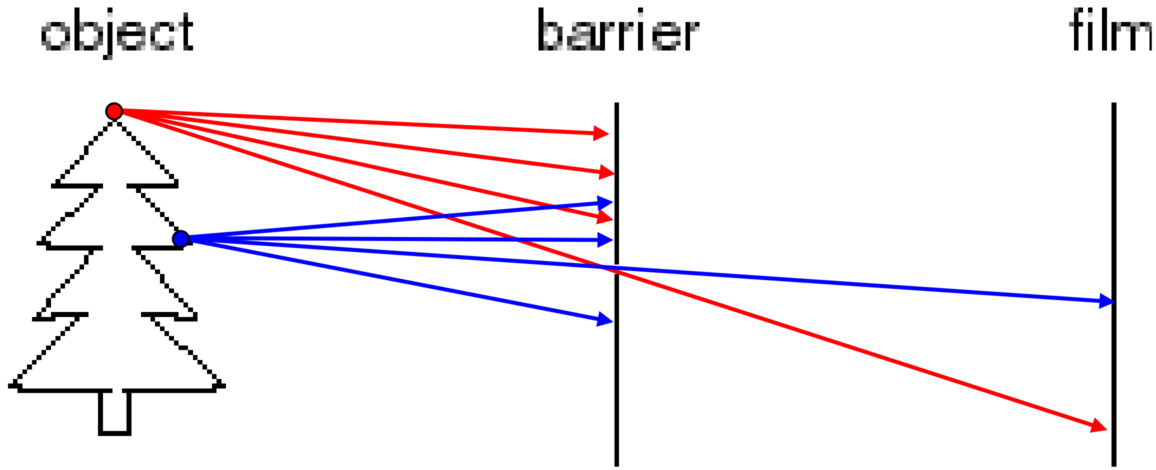


Idea 1: put a piece of film in front of an object

Do we get a reasonable image?

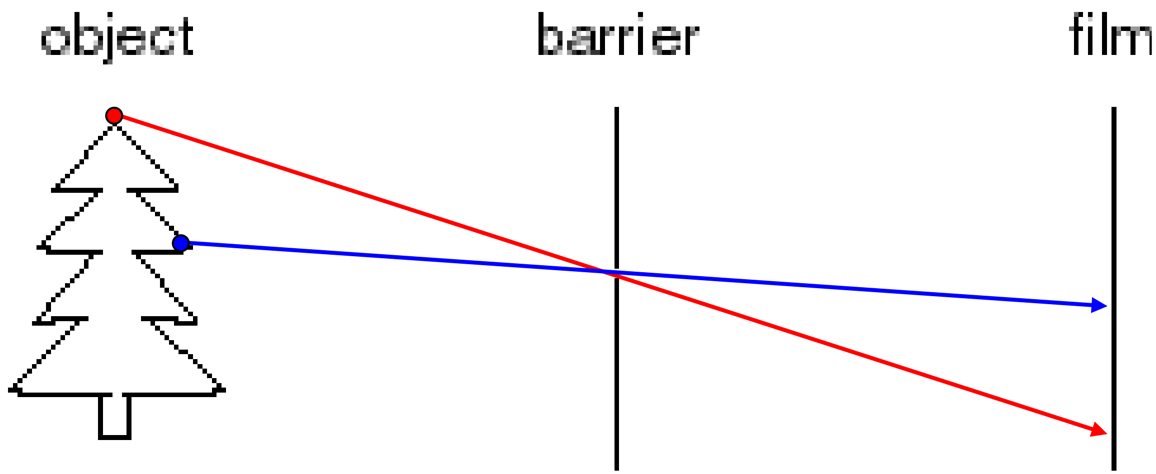
Slide by Steve Seitz

Pinhole camera



Add a barrier to block off most of the rays

Pinhole camera



- Captures **pencil of rays** – all rays through a single point: **aperture**, **center of projection**, **focal point**, **camera center**
- The image is formed on the **image plane**

Slide by Steve Seitz

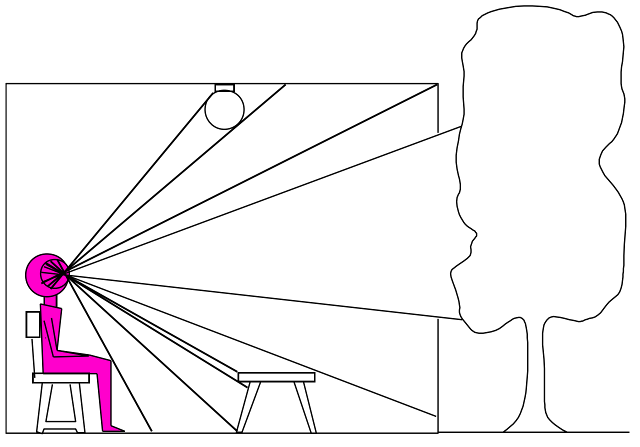
Pinhole cameras abound…

Pinhole images during a solar eclipse

Slide by A.C. Berg

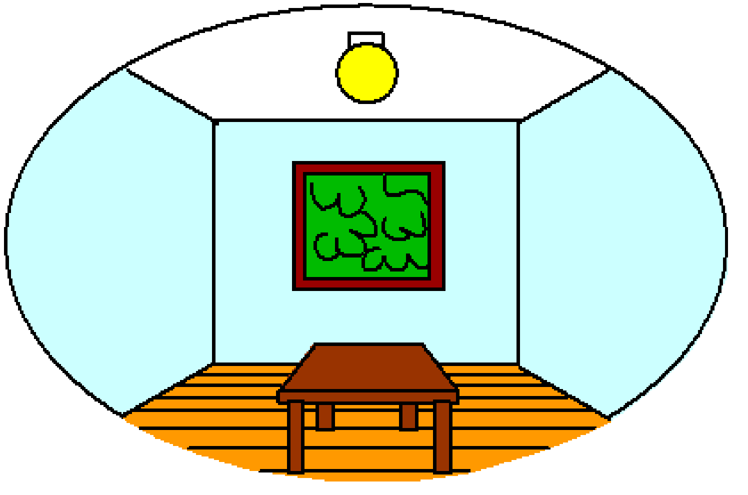
Dimensionality reduction: from 3D to 2D

3D world



Point of observation

2D image



What is preserved?

- Straight lines, incidence

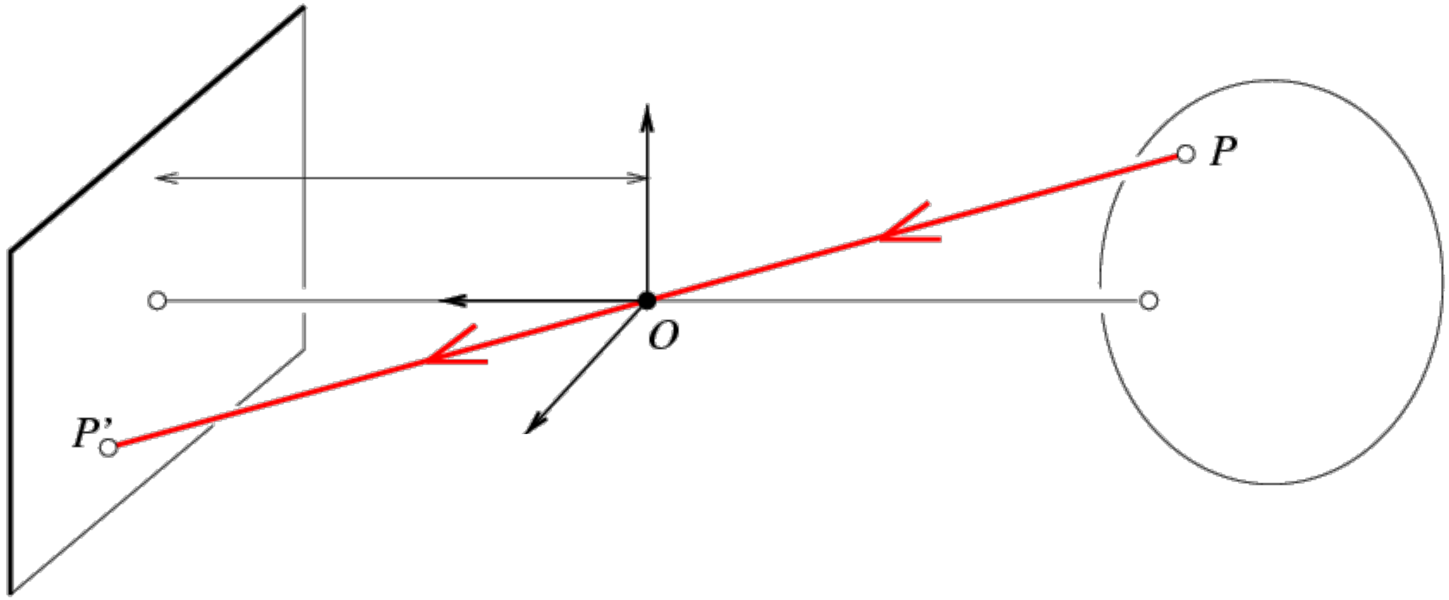
What is not preserved?

- Angles, lengths

Slide by A. Efros

Figures © Stephen E. Palmer, 2002

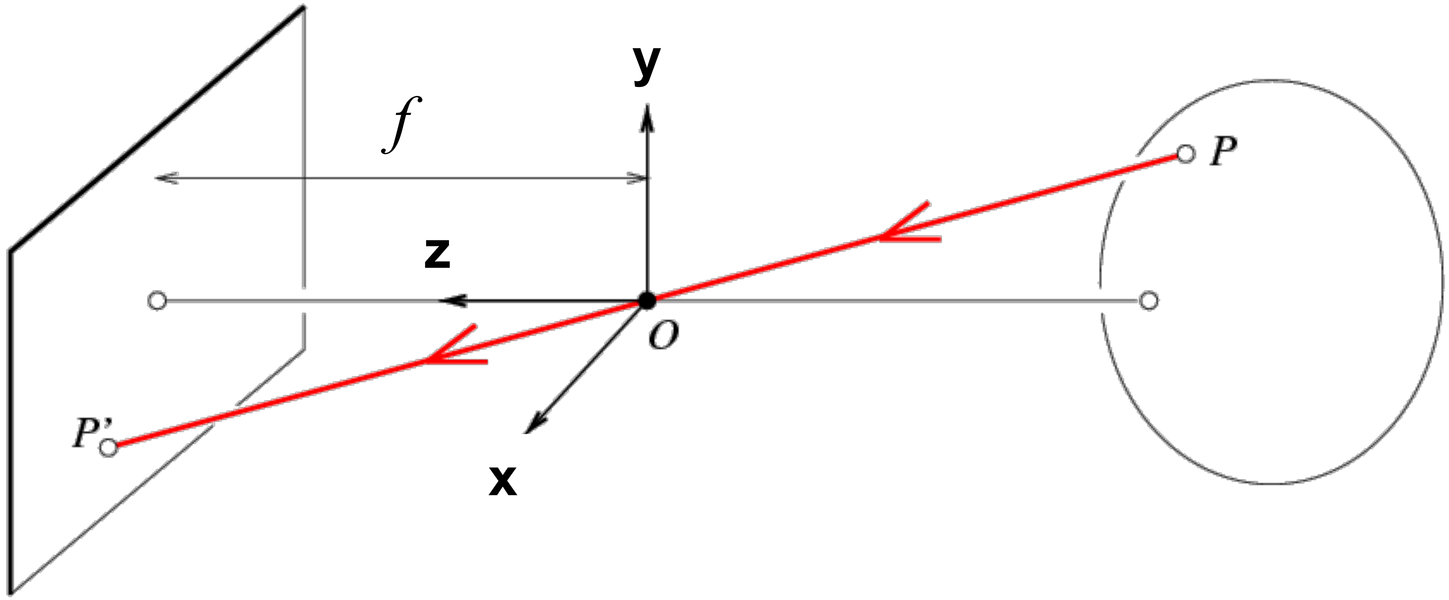
Modeling projection



- To compute the projection *P'* of a scene point *P*, form the **visual ray** connecting *P* to the camera center *O* and find where it intersects the image plane
    - All scene points that lie on this visual ray have the same projection in the image
    - Are there scene points for which this projection is undefined?

Source: J. Ponce, S. Seitz

Modeling projection



The coordinate system

- The optical center (***O***) is at the origin
- The image plane is parallel to xy-plane (perpendicular to z axis)

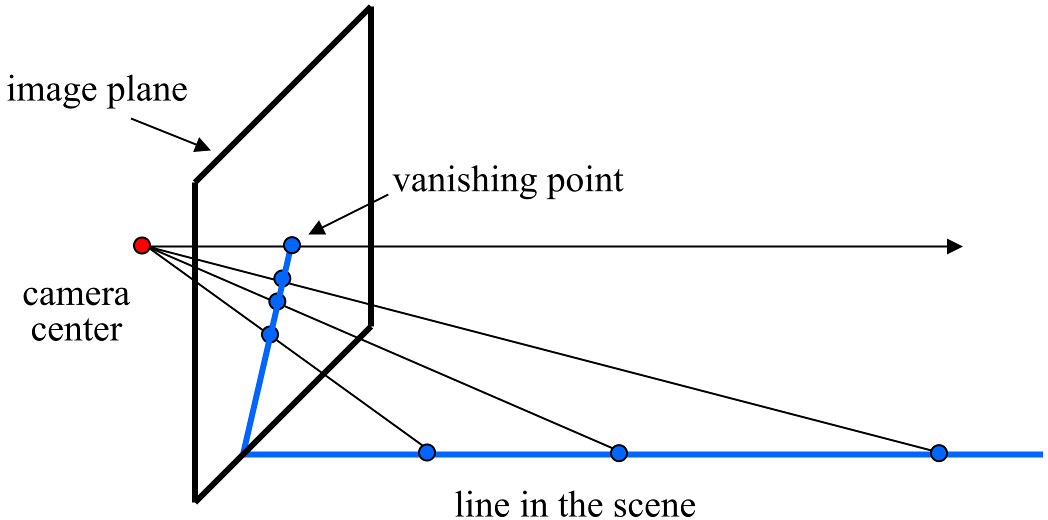
Projection equations

- Derived using similar triangles:

$$(x, y, z) \rightarrow (f\frac{x}{z}, f\frac{y}{z})$$

Source: J. Ponce, S. Seitz

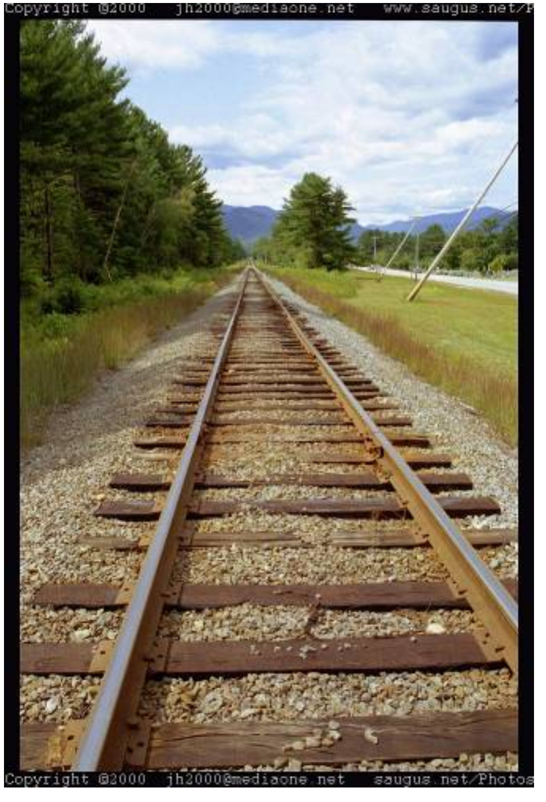
Projection of a line



- What if we have another line in the scene parallel to the first one?

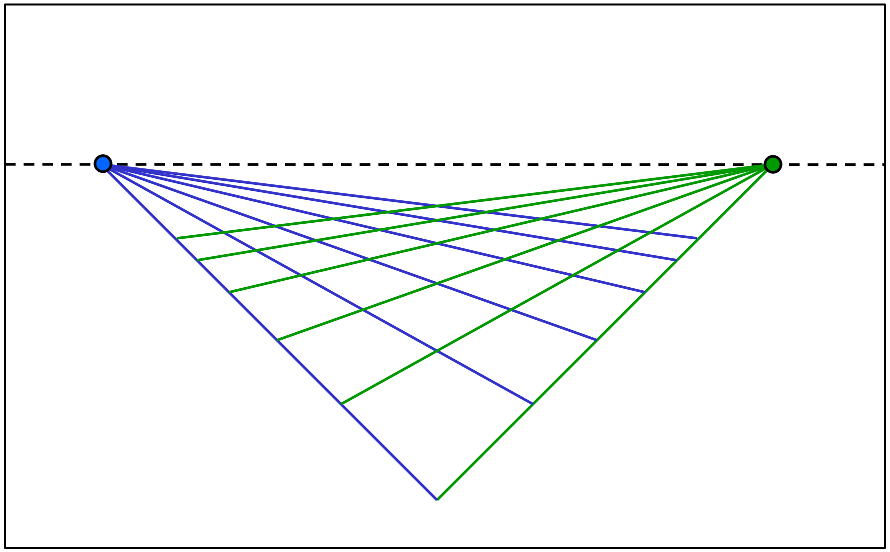
Vanishing points

- Each direction in space has its own vanishing point
  - All lines going in that direction converge at that point
  - Exception: directions parallel to the image plane



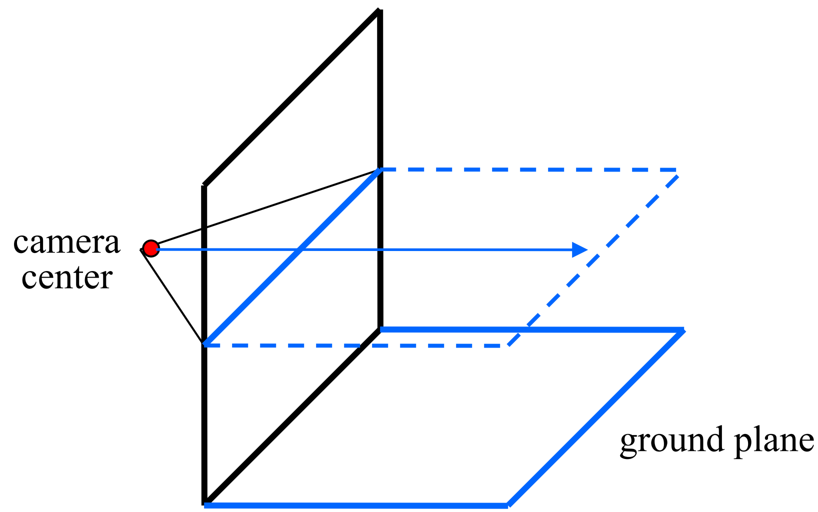
Vanishing points

- Each direction in space has its own vanishing point
  - All lines going in that direction converge at that point
  - Exception: directions parallel to the image plane
- What about the vanishing line of a plane?



Slide by Steve Seitz

The horizon



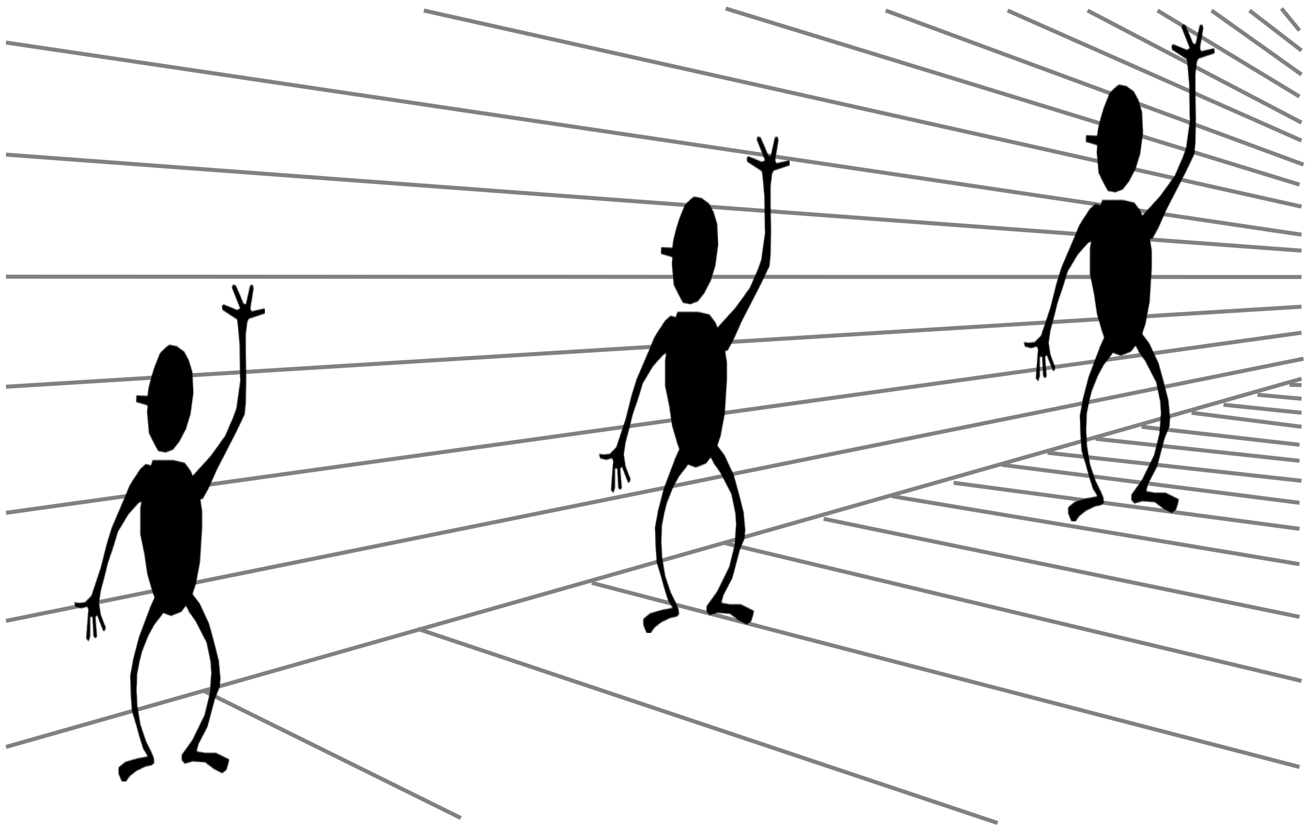
- Vanishing line of the ground plane
  - All points at the same height as the camera project to the horizon
  - Points higher than the camera project above the horizon
  - Provides way of comparing height of objects

Slide by Steve Seitz

The horizon

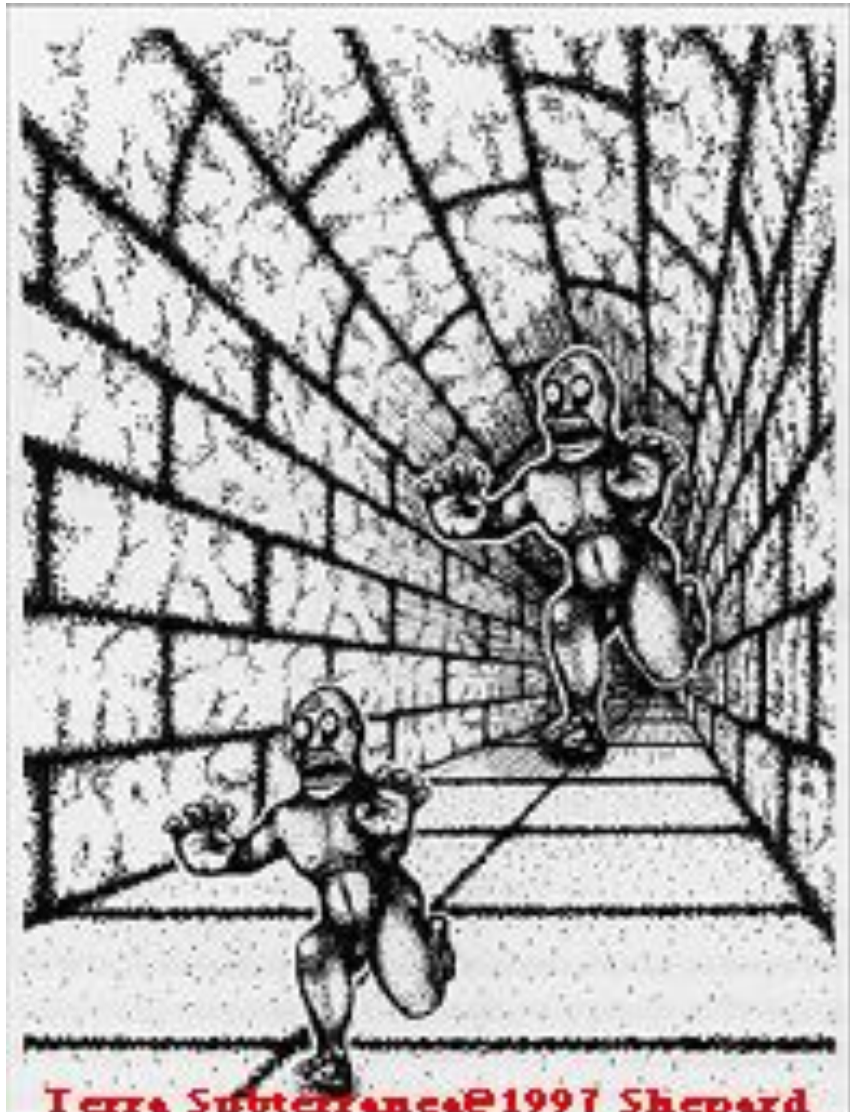
Slide by Steve Seitz

Perspective cues



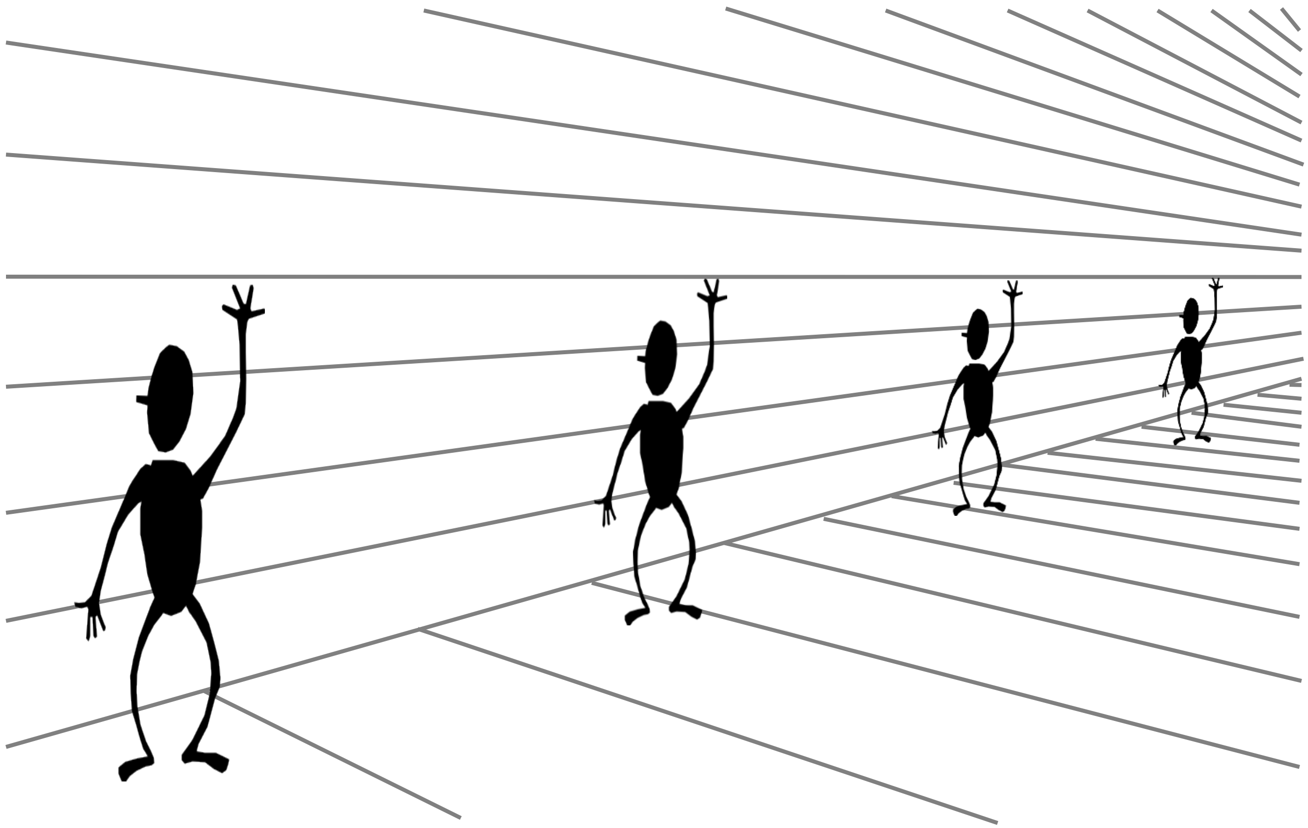
Slide by Steve Seitz

Perspective cues



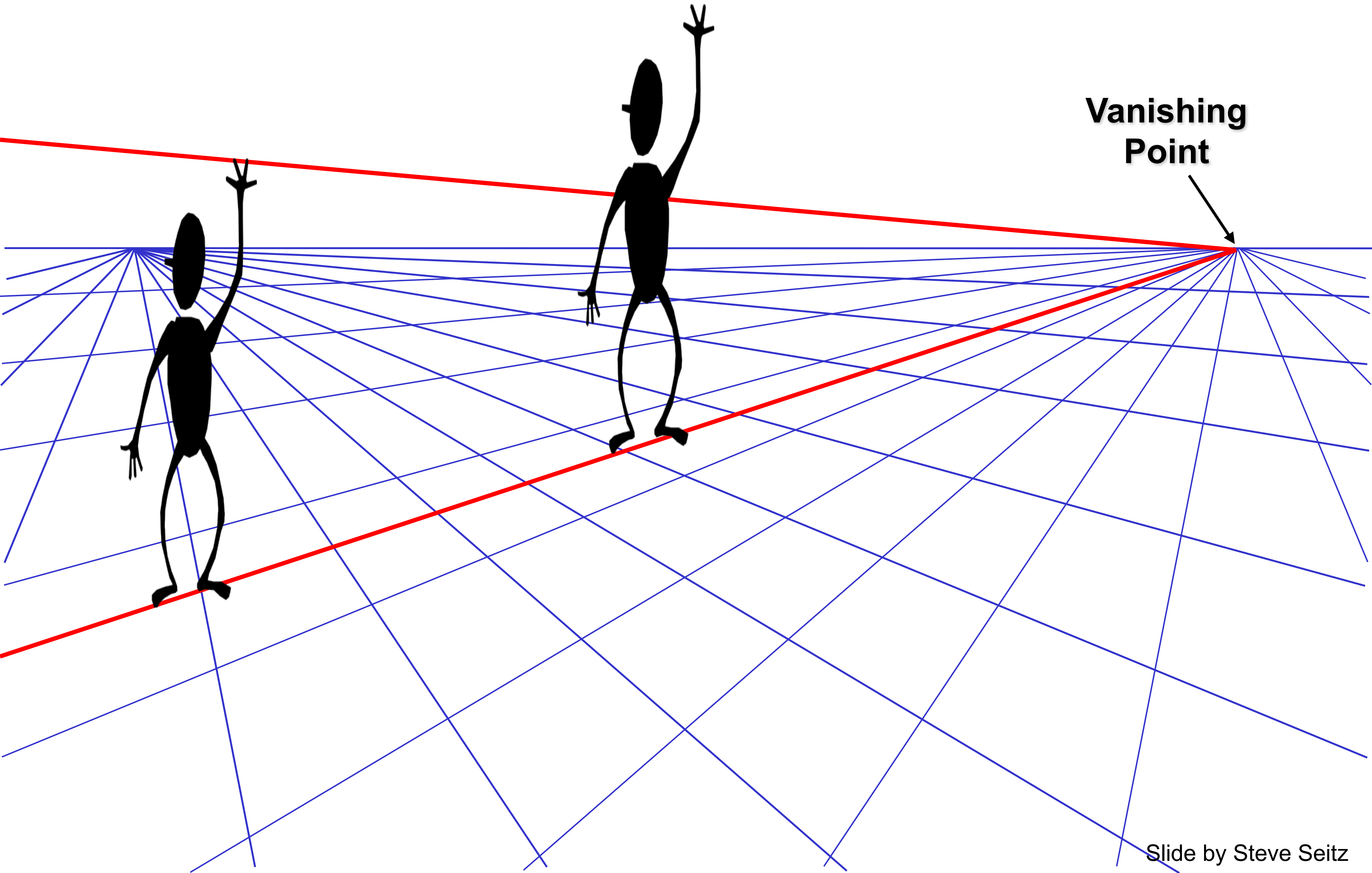
Slide by Steve Seitz

Perspective cues



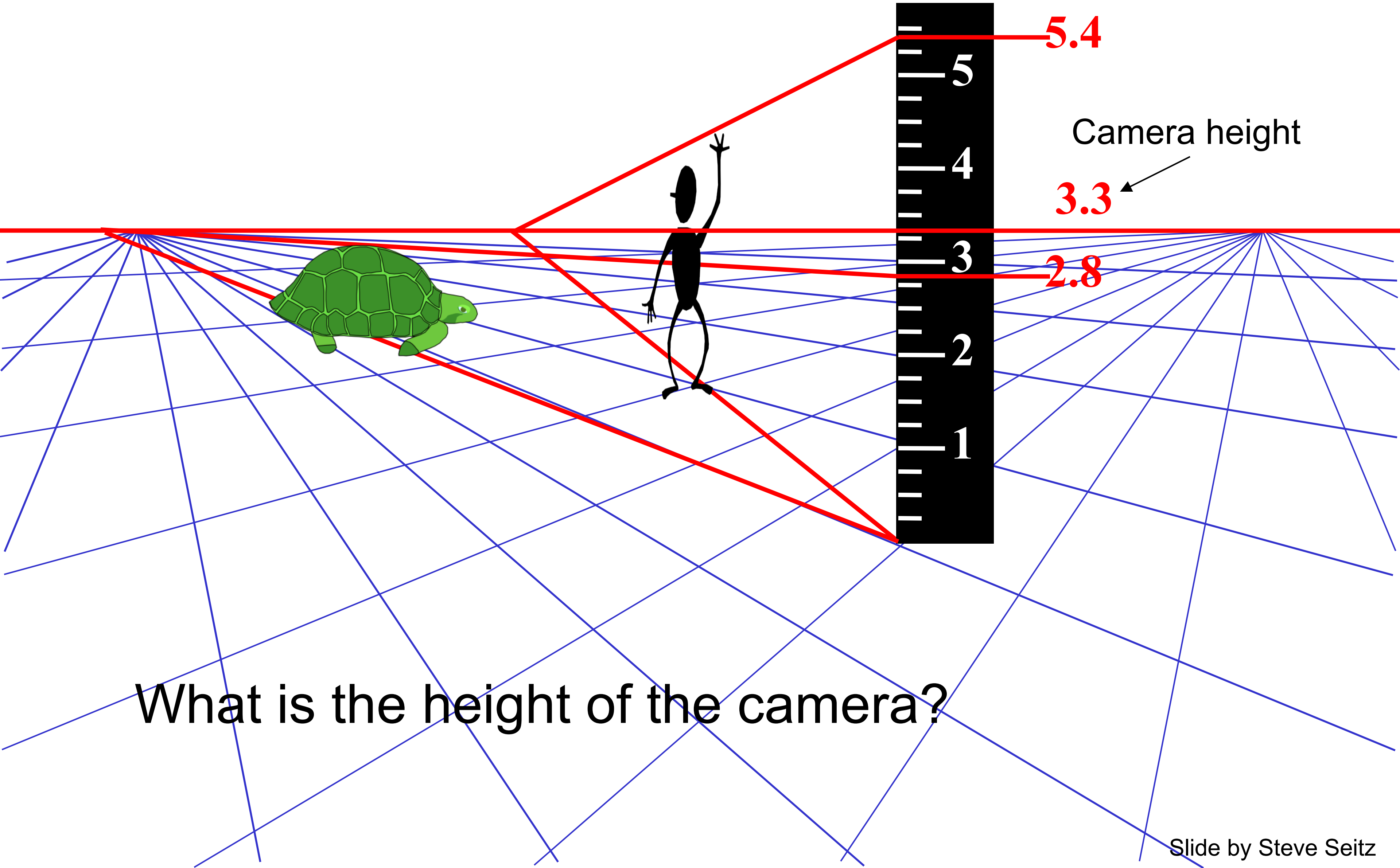
Slide by Steve Seitz

Comparing heights



Slide by Steve Seitz

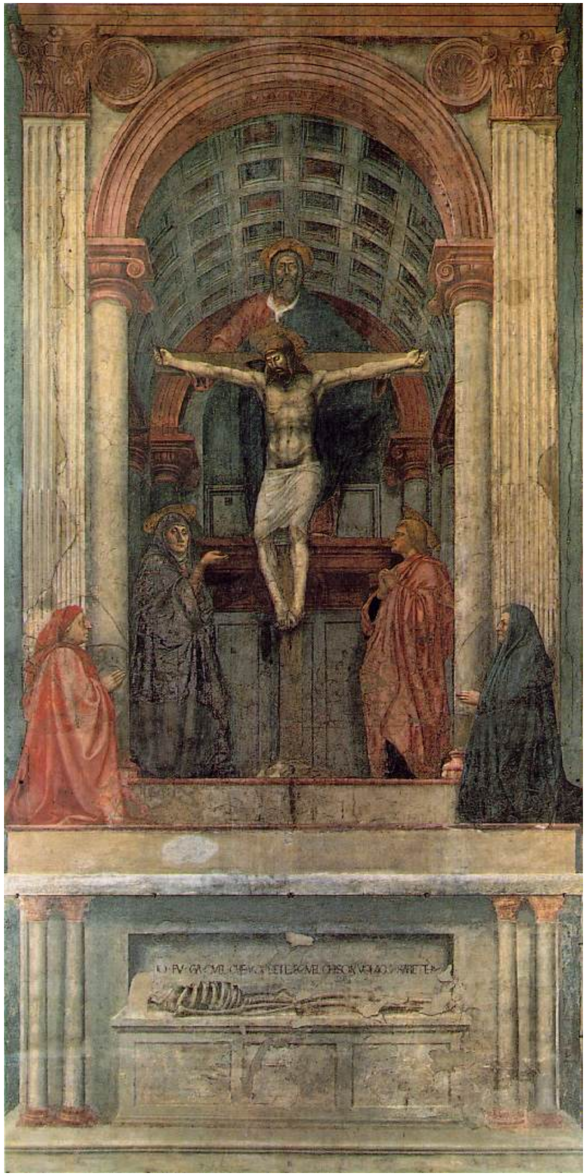
Measuring height

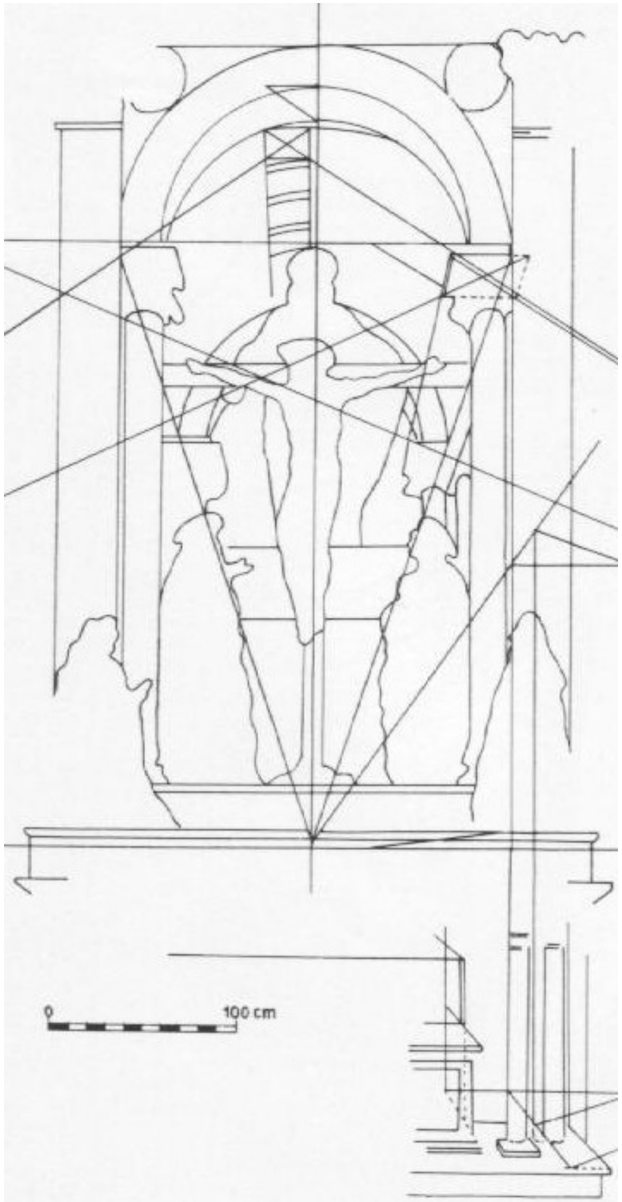


What is the height of the camera?

Slide by Steve Seitz

Perspective in art





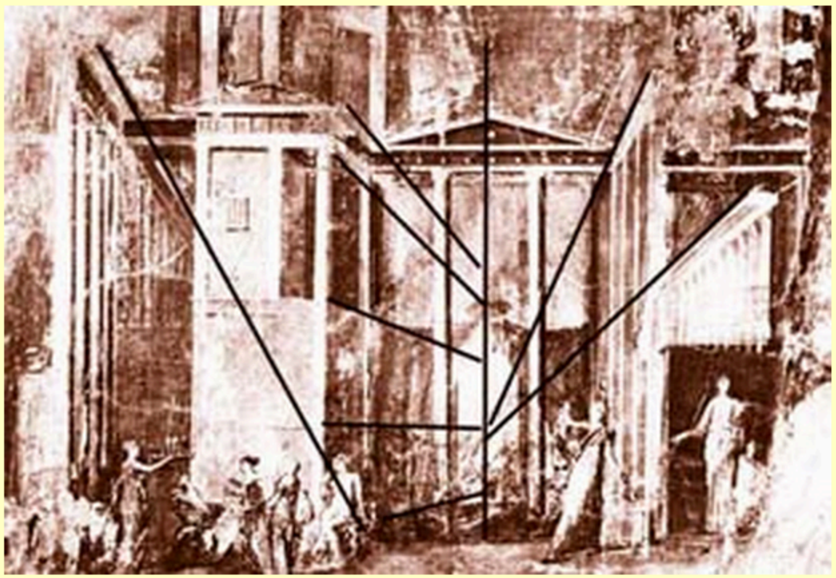
Masaccio, *Trinity*, Santa Maria Novella, Florence, 1425-28

One of the first consistent uses of perspective in Western art

Slide Svetlana Lazebnik

(at least partial) Perspective projections in art well before the Renaissance

Several Pompei wallpaintings show the fragmentary use of linear perspective:

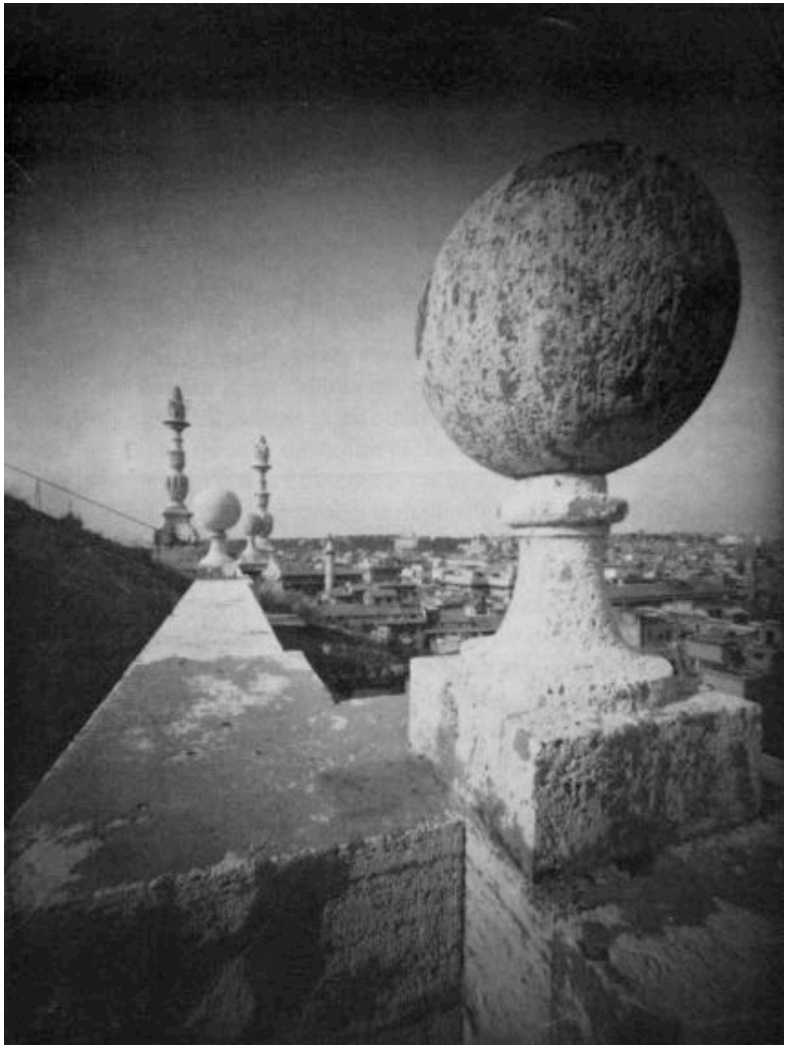


From ottobwiersma.nl

Also some Greek examples,
So apparently pre-renaissance…

Perspective distortion

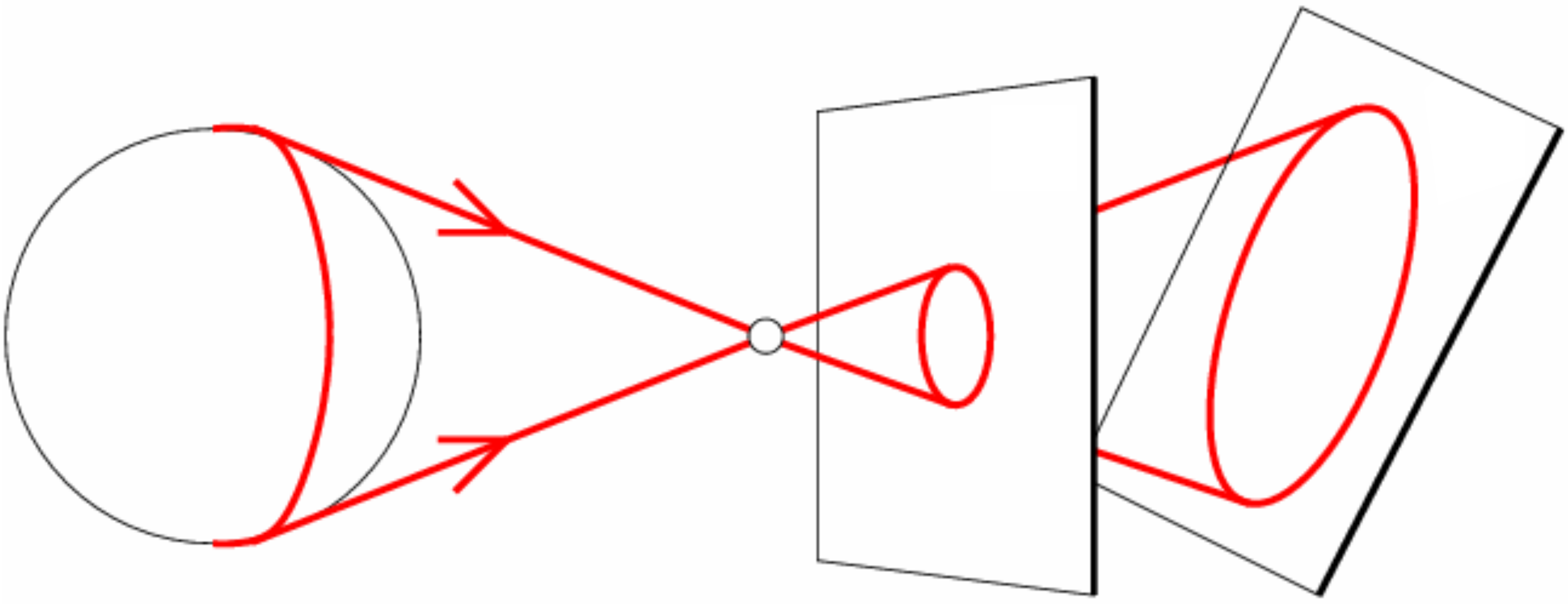
- What does a sphere project to?



M. H. Pirenne

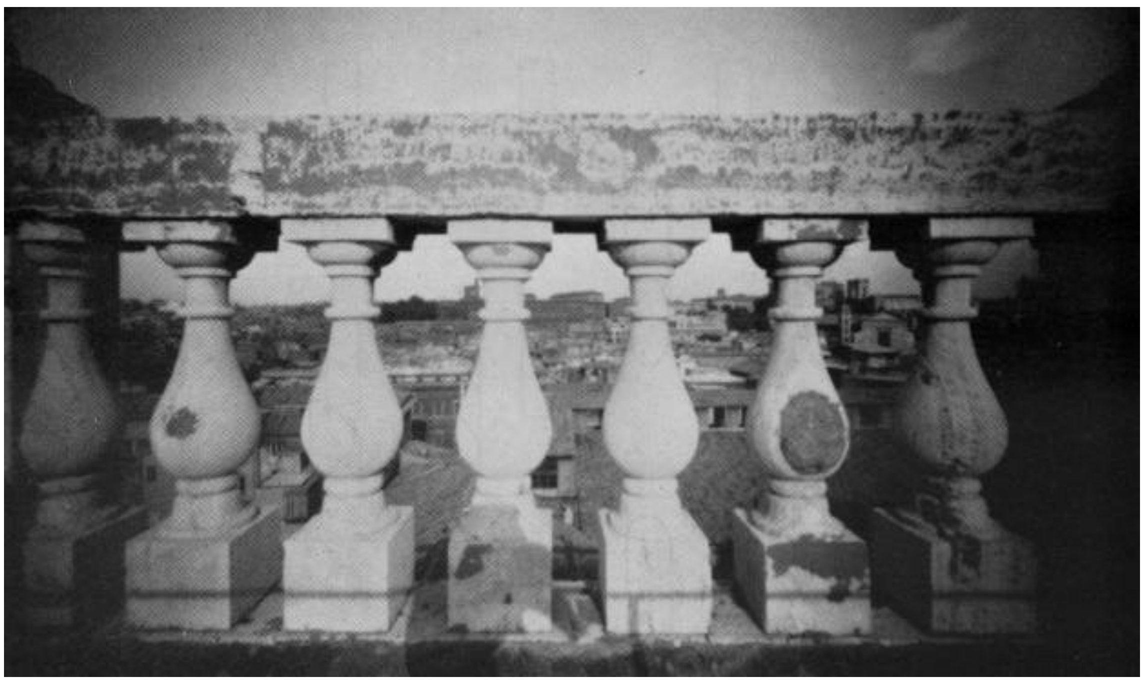
Perspective distortion

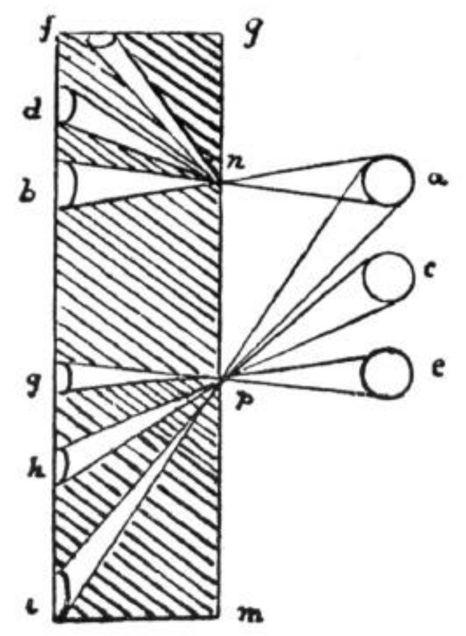
- What does a sphere project to?



Perspective distortion

- The exterior columns appear bigger
- The distortion is not due to lens flaws
- Problem pointed out by Da Vinci

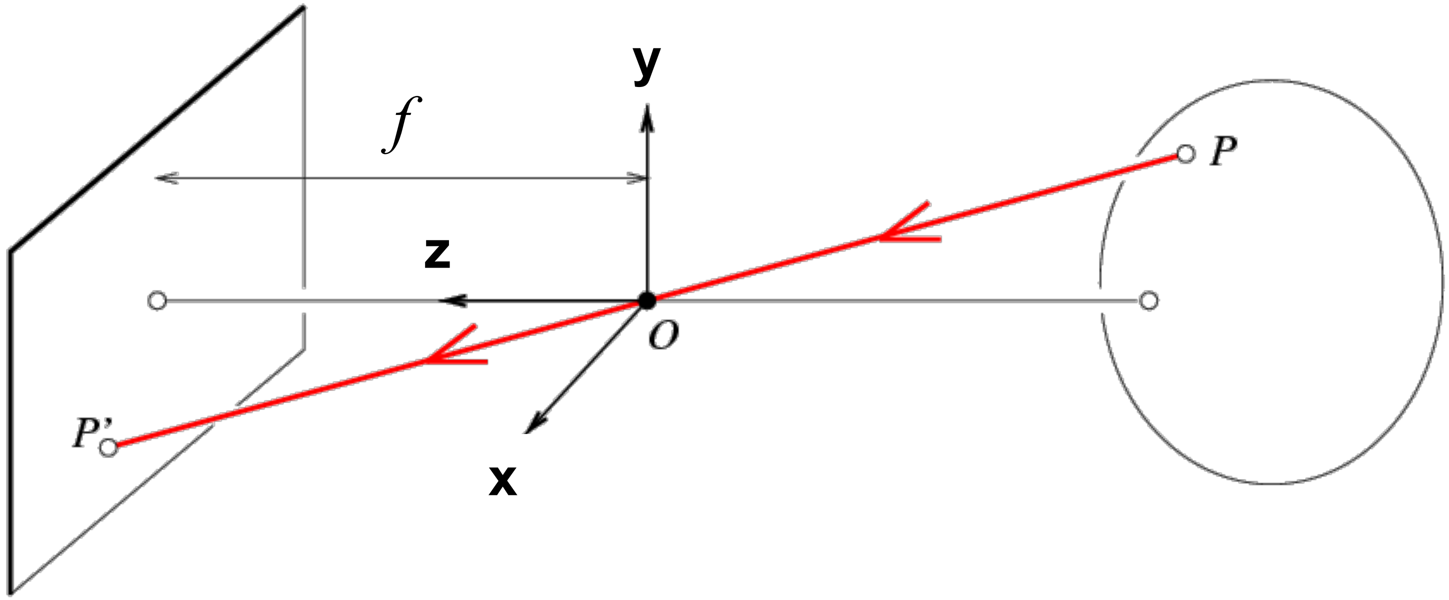




Slide by F. Durand

Perspective distortion: People

Modeling projection



Projection equation: $(x, y, z) \rightarrow (f\frac{x}{z}, f\frac{y}{z})$

Source: J. Ponce, S. Seitz

Homogeneous coordinates

$$(x, y, z) \rightarrow (f\frac{x}{z}, f\frac{y}{z})$$

Is this a linear transformation?

- no—division by z is nonlinear

Trick: add one more coordinate:

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous scene coordinates

Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

Slide by Steve Seitz

Perspective Projection Matrix

Projection is a matrix multiplication using homogeneous coordinates

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z/f \end{bmatrix} \Rightarrow \left(f\frac{x}{z}, f\frac{y}{z}\right)$$

divide by the third coordinate

In practice: lots of coordinate transformations…

$$\begin{pmatrix} \text{2D} \\ \text{point} \\ \text{(3x1)} \end{pmatrix} = \begin{pmatrix} \text{Camera to} \\ \text{pixel coord.} \\ \text{trans. matrix} \\ \text{(3x3)} \end{pmatrix} \begin{pmatrix} \text{Perspective} \\ \text{projection matrix} \\ \text{(3x4)} \end{pmatrix} \begin{pmatrix} \text{World to} \\ \text{camera coord.} \\ \text{trans. matrix} \\ \text{(4x4)} \end{pmatrix} \begin{pmatrix} \text{3D} \\ \text{point} \\ \text{(4x1)} \end{pmatrix}$$

Whole "pipeline"

$$\begin{bmatrix} w_p p_i \\ w_p p_j \\ w_p \end{bmatrix} = \begin{bmatrix} s_x & k_1 & 0 \\ k_2 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f & 0 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\begin{pmatrix} \text{2D} \\ \text{point} \\ \text{(3x1)} \end{pmatrix} = \begin{pmatrix} \text{Camera to} \\ \text{pixel coord.} \\ \text{trans. matrix} \\ \text{(3x3)} \end{pmatrix} \begin{pmatrix} \text{Perspective} \\ \text{projection matrix} \\ \text{(3x4)} \end{pmatrix} \begin{pmatrix} \text{World to} \\ \text{camera coord.} \\ \text{trans. matrix} \\ \text{(4x4)} \end{pmatrix} \begin{pmatrix} \text{3D} \\ \text{point} \\ \text{(4x1)} \end{pmatrix}$$

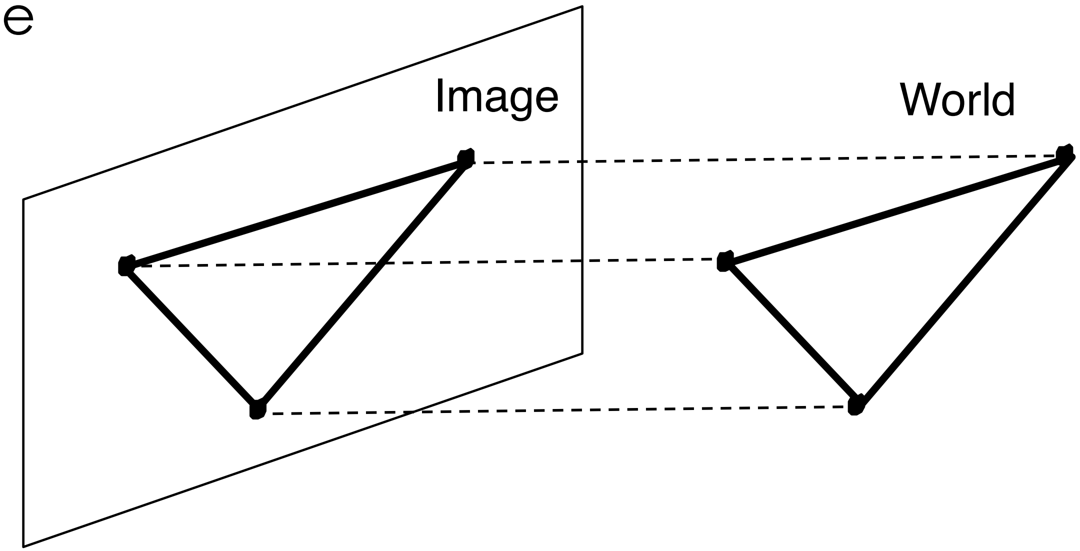
Just one matrix (+ dehomogenization)
but with a special structure

$$\begin{bmatrix} w_p p_i \\ w_p p_j \\ w_p \end{bmatrix} = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Orthographic Projection

Special case of perspective projection

- Distance from center of projection to image plane is infinite



- Also called "parallel projection"
- What's the projection matrix?

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \Rightarrow (x, y)$$

Slide by Steve Seitz

More reading & thought problems

Shape from Chebyshev nets, Koendereink & van Dorn

Accidental pinhole and pinspeck cameras. Torralba & Freeman

Show that a sphere can look like a non-circular under perspective projection.

What does a pinhole camera image look like as you make the pinhole larger?

Sit down and write out some equations for perspective vs parallel projection…